7. Limits

Limits of Exponential and Logarithmic Functions

The following results are useful in the evaluation of the limits of exponential and logarithmic functions:

1. $\lim_{x\to 0}\frac{a^x-1}{x}=\log_e a,\ \ a>0$

2. $\lim_{x\to 0}\frac{e^x-1}{x}=1$

3. $\lim_{x\to 0}\frac{\log(1+x)}{x}=1$

4. $\lim_{x\to 0}(1+x)^{\frac{1}{x}}=e$ **Limits at Infinity**

A function *f* is said to tend to a limit *L* as $x\to\infty$ if for $\delta>0$, however small it may be, there exists a positive number *k* such that $|f(x)-L|<\delta\ \forall\ x$ in the domain of *f*, for which $x > k$. This can be mathematically written as $\lim_{x\to\infty}f(x)=L$.

1. $\lim_{x\to\infty}C=\lim_{x\to-\infty}C=C$, where *C* is a constant.

2. $\lim_{x\to\infty}\frac{C}{x^n}=0,\ n>0$

3. $\lim_{x\to-\infty}\frac{C}{x^n}=0,\ n\in N$

Infinite Limit

If for every $k > 0$, there exists $\delta>0$ such that for all *x* in the domain of *f* and $x\in(a-\delta, a+\delta)$, we have $f(x) > k$, then the limit of $f(x)$ as *x* tends to *a* is infinity. This can be mathematically written as $\lim_{x\to a}f(x)\to\infty$.

The following steps can help evaluate algebraic limits at infinity:

1. If the function is not in rational form, i.e., $\frac{f(x)}{g(x)}$, then first express it in rational form.

2. Divide the numerator and denominator by x^n , where *n* is the highest power of *x*.

3. Use the result $\lim_{x\to\infty}\frac{C}{x^n}=0,\ n>0$, and $\lim_{x\to\infty}C=C$.